

Gravity dual of dynamically broken supersymmetry

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Abstract

We study a renormalization group flow of ABJM theory embedded into the warped A_8 geometry and explore the dependence of the vacuum structure on the parameters of the theory. This model has a product group gauge structure $U(N)_k \times U(n+l)_{-k}$ and comes equipped with discrete parameters N , l , and k , a continuous parameter b_∞ related to the ratio of the Yang-Mills coupling for the two gauge groups, and one dimensionful parameter g_{YM}^2 setting the overall scale. A supersymmetric supergravity solution exists when $Q = N - l(l-k)/2k - k/24$ is positive and is interpretable as a RG flow from a Yang-Mills like UV fixed point to a superconformal IR fixed point with free energy of order $Q^{3/2}$. The fate of the theory when Q is taken to be negative is less clear. We explore the structure of the possible gravity solution for small negative Q by considering the linearized gravitational back reaction from adding a small number of anti-branes on the $Q = 0$ background. Following the work of Bena, Graña, and Halmagyi, we find that a sensible solution satisfying appropriate boundary conditions does not appear to exist. This leaves the status of the RG flow for the $Q < 0$ theories a mystery. We offer the following speculative resolution to the puzzle: the $-k/24$ unit of charge induced by the curvature correction to supergravity should be considered an allowed physical object, and one should be adding an anti brane not to the $Q = 0$ background but rather the $Q = -k/24$ background. Such a solution has a repulson singularity, and gives rise to a picture of the vacuum configuration where a cluster of anti-branes are floating around the repulson singularity, but are stabilized from being pushed off to infinity by other fluxes. Such a state is non-supersymmetric and appears to describe a vacuum with dynamical breaking of supersymmetry. Based on these considerations, we construct a phase diagram for this theory exhibiting various interesting regions.

1 Introduction

Dynamical supersymmetry breaking (DSB) is an important phenomenon in supersymmetric field theories. It is a critical ingredient in model building where one aims to incorporate both the benefits of supersymmetry and the empirical fact that this symmetry is not always manifest. Several concrete models exhibiting DSB are known and are reviewed, for example, in [1]. It would be interesting, nonetheless, to extend the list of examples of models exhibiting DSB and to study this phenomenon from new perspectives such as gauge/gravity correspondence.

The model of Aharony, Bergman, Jafferis, and Maldacena (ABJM) is a promising framework to explore this issue. As a field theory, this model is a Chern-Simons theory with product gauge group and level $U(N)_k \times U(N+l)_{-k}$ with specific matter content and interactions [2,3]. For $k > 1$ this model has $\mathcal{N} = 6$ superconformal symmetry. Its candidate gravity dual is M-theory on $AdS_4 \times S^7/Z_k$ with l units of discrete torsion supported by the orbifold. The free energy of this theory can be inferred from the standard Bekenstein-Hawking analysis and was found to exhibit the familiar scaling [4–6]

$$F = -\frac{2\sqrt{2}}{3}k^2 \left(\frac{Q}{k}\right)^{3/2}, \quad (1.1)$$

where

$$Q = N - \frac{l(l-k)}{2k} - \frac{k}{24}. \quad (1.2)$$

This precise form for Q takes into consideration the Freed-Witten anomaly as well as the contribution from the curvature [5]. The expression (1.1) based on supergravity is the leading approximation in the planar limit $k \gg 1$ as well as in the limit of large 't Hooft coupling

$$\lambda = \frac{Q}{k}. \quad (1.3)$$

In the very remarkable work of [6], the exact λ dependence of the free energy in the leading planar approximation, reproducing the leading large λ dependence (1.1) including the $-k/24$ curvature contribution, was computed on the field theory side using the localization technique of [7]. This is a highly non-trivial test of the gauge gravity correspondence for the ABJM system.

One very interesting feature of (1.1) is its non-analyticity at $Q = 0$. It is certainly possible to find a combination of integer parameters N , l , and k to make $Q < 0$, but for such a value of Q , the free energy ceases to be real. This should be interpreted as an indication that the assumption of superconformal symmetry of the field theory which goes into the localization analysis is breaking down when Q becomes negative. Does this mean that a field theory with

these sets of parameters is intrinsically ill behaved, or does it simply mean that the theory exists in some gapped phase? It is not immediately clear how to address this issue short of solving the theory completely.

One can attempt to study this issue from the perspective of the dual gravity formulation. However, the candidate dual geometry of $AdS_4 \times S^7/Z_k$ with radius Q ceases to exist when Q becomes negative, and it is not clear how one should proceed dealing directly with the gauge gravity duality in the ABJM theory.

One way in which we can gain some perspective on this issue is to consider embedding the ABJM theory in some renormalization group flow where the ultra-violet theory is expected to be well behaved for general values of N , l , and k . One can then study the phase of this theory as we vary these parameters. If the ultra-violet theory also admits a gravity dual, one can explore the phase structure of this theory from the structure of the full gravity solution.

Perhaps the most natural ultra-violet embedding of the ABJM theory is to turn on a Yang-Mills term for each of the gauge groups $U(N)$ and $U(N + l)$. The structure of the gravity dual of this embedding is reasonably well understood [5,8]. Unfortunately, the gravity dual is more complicated in structure than that of a simple cohomogeneity one solutions, making it extremely cumbersome to work with this setup.

Fortunately, there is another ultra-violet embedding of the ABJM theory which is simpler on the gravity side [9], based on M-theory on an eight-dimensional $spin(7)$ holonomy manifold of cohomogeneity one, known as the A_8 geometry, originally constructed by Cvetič, Gibbons, Lu, and Pope [10]. Roughly speaking, the A_8 manifold interpolates between $R^7 \times S^1$ at infinity, to R^8 in the core, with a $U(1)$ symmetry corresponding to translation in the S^1 at infinity. Near the core, this shift rotates the R^8 around the origin in such a way that taking the Z_k orbifold of this $U(1)$ symmetry gives rise to a R^8/Z_k orbifold precisely of the kind which appears in the construction of the ABJM model. In order to completely formulate the gravity dual, we need to know the self-dual 4-form on the eight dimensional A_8 geometry and the warp factor sourced by charges and fluxes. All of these structures have been constructed explicitly in [10] and can be utilized in interpreting these solutions from the point of view of a holographic renormalization group flow [9].

We will primarily work with $k \gg 1$ corresponding to the planar limit. In this limit, the radius of the asymptotic S^1 is small, and it is best to view the supergravity solution using the language of type IIA supergravity.

In order to explore the fate of $Q < 0$ theory in this framework, it is helpful to understand the theory with $Q = 0$, which corresponds to suitably selecting N , l , and k . From the gravity dual perspective, shifting from $Q = 0$ to $Q > 0$ corresponds, roughly, to adding D2-branes.

Along similar lines, one expects that shifting from $Q = 0$ to $Q < 0$ would correspond to adding anti D2-branes to the $Q = 0$ solution. So the problem of understanding the gravity dual of the $Q < 0$ theory appears to involve the study of the gravitational back reaction of anti-D2 branes in a certain warped geometry with fluxes.

In general, this is a cumbersome problem where one must deal with the effects of the branes on fluxes and vice versa which can be complicated. To make the problem more tractable on the first pass, it is convenient to treat the shift away from $Q = 0$ as being small and to work to first order in that perturbation.

In fact, a very similar problem, computing the gravitational back reaction of anti D3-branes to first order around the Klebanov-Strassler background [11], was initiated in the work of Bena, Graña, and Halmagyi [12] and has been followed up in many papers including [13–17].¹ Much of the analysis for the A_8 geometry is similar to these works and so we will largely follow their template. It should also be noted that despite serious efforts on the part of these authors, these papers report on the *non-existence* of the back reacted solution which is consistent with the expected boundary conditions.

Here in this article, we will re-examine the parallel issue in the A_8 setting. There are a few salient features of the A_8 setup which are distinct from the earlier works which we should point out.

1. One of the main motivations for considering the back reaction of anti-branes in the Klebanov-Strassler geometry was to study the candidate for a gravity dual of a meta-stable vacuum. As we will review shortly, the non-supersymmetric configuration in the case of A_8 is expected to correspond to the true vacuum with dynamical supersymmetry breaking. Unlike the meta-stable vacuum which can afford not to exist in a strongly interacting theory, one expects the true vacuum to exist assuming that the field theory exists.
2. Unlike the 3+1 dimensional construction of Klebanov-Strassler, a 2+1 dimensional construction has a simpler UV structure. Instead of the indefinite cascade, the theory crosses over into a super-renormalizable 2+1 dimensional ultra-violet fixed point. This reduces the risk of causing confusion when identifying the parameters of the gravity solution and the field theory in the ultra-violet region. This advantage of 2+1 dimensions was also highlighted in [13, 16].
3. The warped deformed conifold asymptotes near the core to a deformed cone $R^3 \times S^3$ and as such the anti-D3 sources were smeared along the S^3 to simplify the gravity

¹An earlier attempt to study these backgrounds can be found in [18].

problem. It is generally believed that this is not a serious problem. Nonetheless, the fact that all the earlier works fail to find the candidate gravitational back reaction have caused many to question if this smearing is in part the culprit. In the case of A_8 , there will be no room for such a doubt since the geometry of the $Q = 0$ solution has a unique origin where the anti-D2 will naturally sit and preserve the cohomogeneity one structure without any smearing.

Despite all these differences, the conclusion of our perturbative analysis follows the trend set by our predecessors. We, too, find that there are no solution interpretable as the linearized perturbation around $Q = 0$ to describe the $Q < 0$ background.

In light of this finding, we will offer our speculation on how one should think about the ultimate fate of the $Q < 0$ theory.

The organization of this paper is as follows. We will start in section 2 by reviewing the warped A_8 geometry and its interpretation as a gravity dual to a ultra-violet embedding of the ABJM model. We will then review the perturbative analysis around the $Q = 0$ solution following the template of [12] in section 3. We will offer our conclusions and speculations in section 4.

2 Review of the warped A_8 background

In this section, we review the warped A_8 background. We will begin by reviewing the supergravity solution in subsection 2.1. In subsection 2.3, we will review the dynamics of the field theory which can be inferred from the brane description of the theory.

2.1 Supergravity solution

In this subsection, we will recall the essential features of the warped A_8 geometry originally constructed by [10]. Most of what we review can be found in section 4 of [9]. We start by considering an eight dimensional $spin(7)$ holonomy manifold

$$ds_{A_8}^2 = h(r)^2 dr^2 + \ell^2 (a(r)^2 (D\mu^i)^2 + b(r)^2 \sigma^2 + c(r)^2 d\Omega_4) \quad (2.1)$$

where σ^2 and $(D\mu^i)^2$ are line elements of the S^3 fiber on an S^4 base where S^3 itself is viewed as a S^1 fiber over an S^2 base [10]. Functions $a(r)$, $b(r)$, $c(r)$, and $h(r)$ are given by

$$\begin{aligned} h(r)^2 &= \frac{(r + \ell)^2}{(r + 3\ell)(r - \ell)} \\ a(r)^2 &= \frac{1}{4\ell^2} (r + 3\ell)(r - \ell) \end{aligned}$$

$$\begin{aligned}
b(r)^2 &= \frac{(r+3\ell)(r-\ell)}{(r+\ell)^2} \\
c(r)^2 &= \frac{1}{2\ell^2}(r^2 - \ell^2) .
\end{aligned}
\tag{2.2}$$

The parameter ℓ sets the scale of this geometry. Topologically, this space is R^8 . Geometrically, for large r , this geometry has the structure of $R^7 \times S^1$. We will consider orbifolding the coordinate φ so that it is periodic under $4\pi/k$. The fixed r slice of this geometry has the topology of a squashed S^7/Z_k which can also be viewed as a $U(1)$ bundle over a squashed CP^3 .

The self-dual 4-form on this geometry is also known explicitly. It is given by

$$G_4 = dC_3 \tag{2.3}$$

where

$$C_3 = mB_{(3)} + wd\sigma \wedge d\varphi \tag{2.4}$$

and

$$B_{(3)} = v_1(r)\sigma \wedge X_{(2)} + v_2(r)\sigma \wedge Y_{(2)} + v_3(r)Y_{(3)} \tag{2.5}$$

with

$$v_1(r) = -\frac{(r-\ell)^2}{8(r+\ell)^2} \tag{2.6}$$

$$v_2(r) = \frac{(r-\ell)^2(r+5\ell)}{8(r+\ell)(r+3\ell)^2} \tag{2.6}$$

$$v_3(r) = -\frac{(r-\ell)^2}{16(r+3\ell)^2} \tag{2.7}$$

as is given in [10]. We have also included a locally exact term proportional to w which turns out to play an important role in quantizing the charges.² m is an adjustable parameter for the time being.

We consider embedding this eight dimensional space in M-theory. The resulting geometry will have 2+1 Poincaré symmetry. The self-dual 4-form embeds naturally as components of the M-theory 4-form field strength. It sources the M-theory 4-form electrically through the equation

$$d *_1 F_4 = \frac{1}{2} F_4 \wedge F_4 + (2\pi l_p^{11})^6 k Q \delta^8(\vec{r}) . \tag{2.8}$$

where we included the possibility of additional charge source parametrized by Q . All of the M-theory equations can be solved by the ansatz

$$ds^2 = H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3} ds_{A_8}^2$$

²In [9], w was referred to as α .

$$F_4 = dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1} + G_4 \quad (2.9)$$

with

$$H(r) = \frac{24\pi^2(l_p^{11})^6 k Q}{\ell^6} H_1(r) + \frac{m^2}{\ell^6} H_2(r) \quad (2.10)$$

$$H_1(r) = \frac{\ell(3r^3 - 3r^2\ell - 11r\ell^2 + 27\ell^3)}{192(r - \ell)^3(3\ell + r)} + \frac{1}{256} \log\left(\frac{r - \ell}{r + 3\ell}\right) \quad (2.11)$$

$$H_2(r) = \frac{\ell^5(63\ell^2 + 26r\ell + 3r^2)}{20(r + \ell)^2(r + 3\ell)^5} \quad (2.12)$$

We have anticipated taking the $\alpha' \rightarrow 0$ decoupling limit and dropped the “1” term in $H(r)$. So as $r \rightarrow \infty$, $H \rightarrow 0$. For $k \gg 1$, it is convenient to work in the type IIA description by reducing along the φ coordinate. In the IIA reduction, the M-theory 3-form C_3 gives rise to a IIA NSNS B_2 field given by

$$B_2 = \frac{2}{kR} m(r - \ell)^2 \left(-\frac{1}{8(r + \ell)^2} X_{(2)} + \frac{(r + 5\ell)}{8(r + \ell)(r + 3\ell)^2} Y_{(2)} \right) + \frac{2}{kR} w(X_{(2)} - Y_{(2)}) . \quad (2.13)$$

where

$$R = g_s l_s = \frac{2\ell}{k} \quad (2.14)$$

is the radius of the M-theory circle and

$$l_p^{11} = g_s^{1/3} l_s . \quad (2.15)$$

In order to identify the proper quantization of parameters Q , m , and w which we have introduced in this solution, we need to compute the D2 and the D4 Page charge following [9]. This gives rise to

$$\begin{aligned} Q &= N_2 - \frac{l(l - k)}{2k} - \frac{k}{24} \\ m &= -(4\pi g_s l_s^3) M \\ (2\pi)^2 w &= -(2\pi l_s)^3 g_s \left(l - \frac{k}{2} \right) \end{aligned} \quad (2.16)$$

where

$$M = l - \frac{k}{2} + b_\infty k \quad (2.17)$$

and

$$b_\infty = \frac{1}{(2\pi l_s)^2} \int_{CP^1} B(r = \infty) \quad (2.18)$$

is the period of the NSNS 2-form B through the CP^1 cycle of the CP^3 at $r = \infty$ and is one of the parameters of the background.

With this quantization condition, we find that the D2 Maxwell charge is given by

$$Q_2^{Maxwell} = \frac{1}{g_s(2\pi l_s)^3} \int_{CP^3} r^6 (-H'(r)) \Big|_{r=\infty} = Q + \frac{M^2}{2k} \quad (2.19)$$

Q , on the other hand, is immediately interpretable as the brane charge.

In order to take the $l_s \rightarrow 0$ limit, we scale g_s and r keeping

$$U = \frac{r}{l_s^2}, \quad g_{YM}^2 = \frac{g_s}{l_s} \quad (2.20)$$

fixed as usual. b_∞ is interpreted, as usual, as providing the gauge coupling

$$\frac{1}{g_{YM1}^2} = \frac{b_\infty}{g_{YM}^2}, \quad \frac{1}{g_{YM2}^2} = \frac{(1 - b_\infty)}{g_{YM}^2} \quad (2.21)$$

We have therefore arrived at a family of supergravity solutions, parametrized by N , l , and k which are discrete dimensionless parameters, b_∞ which is a continuous dimensionless parameter, and g_{YM}^2 being the one dimensionful parameter setting the scale of the problem. For fixed k and b_∞ , one can parametrize the remaining choice of models in terms of a set of parameters (Q, M) in lieu of (N, l) .

Note that Q and M are invariant under the transformation

$$N_2 \rightarrow N_2 + l, \quad l \rightarrow l + k, \quad b_\infty \rightarrow b_\infty - 1. \quad (2.22)$$

This is the manifestation of large gauge transformation outlined in [5] and the invariance of Q and M is the result of gauge invariance of physical quantities such as the brane charge and the Maxwell charge.

2.2 Properties of the warped A_8 solution

In this subsection, let us review some of the basic features of the warped A_8 solution.

The solution is parametrized by three discrete parameters N , l , and k , a continuous parameter b_∞ , and a scale g_{YM}^2 . It is convenient to work with some fixed b_∞ . We will also take k to be large so that the planar approximation is reliable. Finally, we will scale

$$N = xk, \quad l = yk \quad (2.23)$$

so that in the large k limit, x and y can be viewed as continuous parameters washing out the granularity of the integers N and l . In terms of x and y , we can express

$$\frac{Q}{k} = x - \frac{y(y-1)}{2} - \frac{1}{24}, \quad \frac{M}{k} = y - \frac{1}{2} + b_\infty. \quad (2.24)$$

The Maxwell charge can then be expressed as

$$\frac{Q_2^{Maxwell}}{k} = \frac{Q}{k} + \frac{M^2}{2k^2} = x + \left(y - \frac{1}{2}\right) b_\infty + \frac{1}{2} b_\infty + \frac{1}{12} . \quad (2.25)$$

This is the quantity which we need be large in order suppress the α' corrections at least for a large part of the bulk region in the gravity dual.

The physical characteristic of this supergravity background depends sensitively on the sign of Q .

For positive Q , the solution asymptotes to $AdS_4 \times S^7/Z_k$ in the deep IR region. This is the dual gravity description of the ABJM superconformal fixed point. As one flows up the holographic renormalization group flow, the period³

$$b(r) = \frac{1}{(2\pi l_s)^2} \int_{CP^1} B(r) \quad (2.26)$$

runs from

$$b(r = \ell) = -\frac{l}{k} + \frac{1}{2} \quad (2.27)$$

to

$$b(r = \infty) = b_\infty . \quad (2.28)$$

Each time $b(r)$ goes outside the range $0 < b(r) < 1$, one can apply the large gauge transformation (2.22) to bring it back into that range. The resulting change in N and l is the manifestation of the duality cascade. Unlike the case of the deformed conifold where the cascade continues forever, here $b(r)$ approaches a limiting value b_∞ after undergoing finitely many cascades. The formula for the gauge coupling (2.21) makes sense only in the gauge where $0 < b_\infty < 1$.

Consider now the case where $Q = 0$, while keeping $Q_2^{Maxwell} > 0$. The structure of the solution is not so dramatically changed in the ultra-violet region. However, in the infra-red region, the solution looks very different. The term proportional to Q in the warp factor (2.10) is gone, and $H(r)$ approaches a finite value as r approaches ℓ . This means that the geometry is regular at $r = \ell$. This is suggestive of the field theory exhibiting a mass gap. One simple way to holographically estimate the mass gap is to compute the time, in field theory coordinate, for a light signal to travel from the boundary to $r = \ell$ [19]

$$t_{gap} = \int_\ell^\infty \frac{dt}{dr} dr = \int_\ell^\infty \sqrt{-\frac{g_{rr}}{g_{tt}}} dr = \int_\ell^\infty H^{1/2}(r) h(r) dr \sim \frac{|M|}{g_{YM}^2 k^2} \quad (2.29)$$

³Note that the period $b(r)$ is not the same as the function $b(r)$ in the ansatz (2.1) for the A_8 geometry. Hopefully, it is clear from the context which $b(r)$ we are talking about.

from which we can read off the scale

$$E_{gap} \sim \frac{1}{t_{gap}} \sim \frac{g_{YM}^2 k^2}{|M|} \sim \frac{g_{YM}^2 k^{3/2}}{N^{1/2}} \quad (2.30)$$

where in the last relation, we used $N \gg k$, $Q = 0$, and assumed b_∞ is of order one.

Finally, consider what happens when Q is taken to be negative. Now, we see that $H_1(r)$ and $H_2(r)$ contribute to $H(r)$ with opposite signs in (2.10). Since $H_1(r)$ diverges at $r = \ell$, we learn that the background exhibits a naked singularity when $H(r)$ becomes zero at some value of $r > \ell$. Presumably, this singularity stems from extrapolating the background supported by positive charge, positive tension BPS sources to negative charge and negative tension. Since negative tension objects are unphysical, what one must do to continue beyond the $Q = 0$ background to negative Q is to add a positive tension negative charge object, i.e. an anti D2-brane.

We have therefore arrived at a conclusion that in order to explore the physics of $Q < 0$ model, we must consider the gravitational back reaction from adding an anti D2-brane to the $Q = 0$ background. In this sense, the problem is very similar to the program of [12].

2.3 Field theory dynamics

Before proceeding to analyze the gravitational back reaction of anti D2-branes let us review our expectation from the field theory considerations.

One disadvantage of the ultra-violet embedding based on A_8 as opposed to turning on the Yang-Mills coupling for the ABJM theory is the fact that the field theory dual of the A_8 construction is not known at the same level of detail. For example, the precise form of the Lagrangian defining the field theory dual has not been written down. Nonetheless, one can infer quite a lot from the form of the background on the gravity side, as well as from the consideration of the associated brane construction.

The warped A_8 supergravity solution asymptotes to a squashed CP^3 cone in type IIA theory warped by $Q_2^{Maxwell}$ units of electric four form flux through CP^3 with b_∞ unit of B_2 on the unique CP^1 cycle of the CP^3 . So the dual field theory appears to resemble a Yang-Mills theory with a product gauge group which is superrenormalizable, and the gravity description is taking over as the effective description below the energy of order [20]

$$E = g_{YM}^2 Q_2^{Maxwell} . \quad (2.31)$$

The supergravity solution also suggests that the theory flows to ABJM in the infra red. This suggests, just as in the case of the ABJM theory, that this model can be engineered

as a decoupling limit of a Hanany-Witten like construction of branes stretched between overlapping 5-branes separated along a compact dimension [2, 3, 5]. We also know from the structure of the supergravity solution that this background preserves $\mathcal{N} = 1$ supersymmetry in the 2+1 dimensional sense [10]. A catalog of overlapping 5-brane configuration, which we will refer to as the KOO table, was presented by Kitao, Ohta, and Ohta in Table 1 of [21]. In the classification of the KOO table, the ABJM construction appears to correspond to the item 4-(iii). In contrast, the natural candidate dual of the A_8 construction is 4-(i).

The brane construction provides a natural interpretation of the parameter b_∞ as well as the structure of the large gauge transformation (2.22). The b_∞ parametrizes the distance between the 5-branes, which runs as a result of the brane bending effect [22] but asymptotes to a fixed value at large separation. The integer N can be interpreted as the number of the integer D3-branes winding all the way around the compact direction separating the 5-branes, and l is the number of fractional D3-branes stretching between the D5-branes. The large gauge transformation (2.22) can then be seen as corresponding to the way in which N , l , and b_∞ transform as one gradually slides b_∞ by one, causing one of the 5-branes to circumnavigate the compact direction, undergoing Hanany-Witten transitions when the two 5-branes cross. These phenomena are reviewed in [5, 9].

In the framework of brane construction, it is relatively easy to see the difference between the cases when $Q/k \gg 0$ and $Q/k \ll 0$. For $Q/k \gg 0$, N will transform in n cascade steps to

$$N \rightarrow N + nl + \frac{n(n-1)}{2}k \geq Q - \frac{k}{12} \quad (2.32)$$

and so as long as $Q/k > 1/12$, then N is positive definite. But as Q gets smaller, we will encounter a duality cascade where N can become negative. An example where this happens is illustrated in figure 1.

What has been previously noted in [8] is that the configuration, illustrated in figure 1.b, corresponds to a non-BPS stable configuration found in [23] by balancing the repulsive force experienced by the D3 segments and the attractive force arising from the angle of the 5-branes forcing the D3-brane to get longer as they move apart. Schematically, one expects the stable, non-BPS configuration to look like what is illustrated in figure 2.

This brane configuration suggests that the vacuum configuration breaks supersymmetry. Strictly speaking, one should view this claim as being valid only for the brane theory and whether or not this feature survives the field theory limit $\alpha' \rightarrow 0$ needs to be examined closely. One of the goals of this paper is to examine this issue for the decoupled theory by looking at the gravity dual.

Much of what we described so far is very similar to the construction of metastable vacua

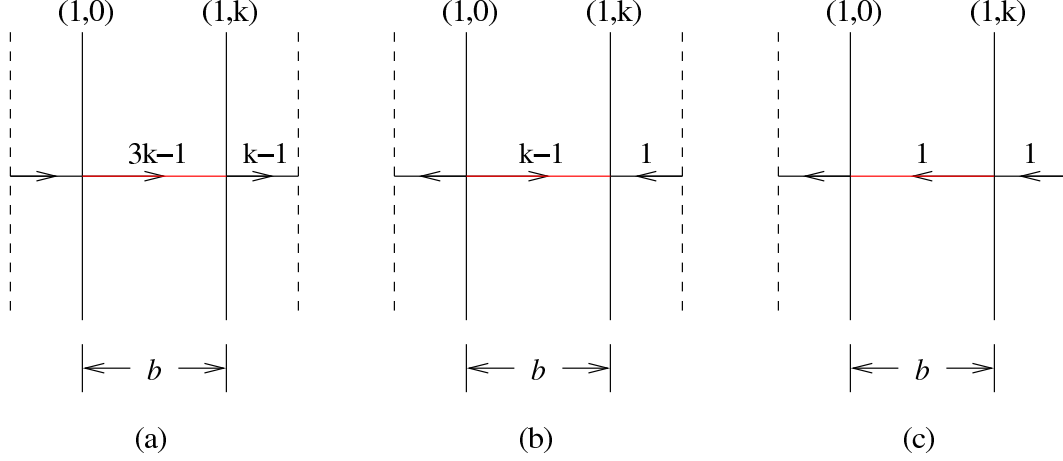


Figure 1: Hanany-Witten brane diagram for configurations violating the generalized s -rule. The configurations (a), (b), and (c) are related by sliding the $(1, k)$ brane around the circle. In this figure, labels such as “ $3k - 1$ ” and “ $k - 1$ ” refers to the number of D3 brane segments stretched between the 5-branes, as opposed to the counting of integer and fractional branes. The configuration (a) corresponds to $N = k - 1$ and $l = 2k$. Configuration (b) corresponds to $N = -1$ and $l = k$. (c) corresponds to $N = -1$ and $l = 0$. This figure originally appeared in [9].

which was the motivation of the work of [12] for the case of the deformed conifold. In that case, the decay channel to the supersymmetric ground state has been identified [24] and the stable supersymmetric vacuum to which the system decays is easy to construct.

In the case of the A_8 background, no comparable decay mechanism or alternate supersymmetric geometry with the same Page charges appear to exist. Therefore, if we were to find the non-supersymmetric supergravity solution with the appropriate charges, it is natural to interpret it as the gravity dual of a vacuum having undergone dynamical supersymmetry breaking.

3 Linearized analysis of non-supersymmetric perturbations

In this section, we will describe the analysis of linearized perturbations around the $Q = 0$ background. Our goal is to identify the linearized perturbation corresponding to adding a small number of anti D2-branes to the $Q = 0$ background. A very similar problem has been analyzed in [13] and [16]. Reference [16] in fact considers a closely related background also by Cvetič, Gibbons, Lu, and Pope [25]. It is therefore extremely convenient to follow the template of the analysis of [16] for our background. The main difference between A_8 and the background of [25] is that the former asymptotes to R^8 whereas the later asymptotes to $R^5 \times S^3$ in the core region. Also, the former, in the IIA description, includes the D6 charge

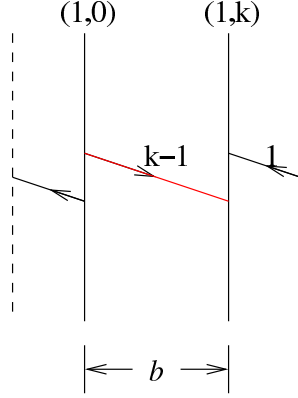


Figure 2: Schematic sketch of the expected minimum energy configuration for the construction illustrated in figure 1.b including the effect of repulsion between the brane segments. This figure originally appeared in [9].

k in addition to the integer D2 charge N and the fractional D2 (integer D4) charge l . The holonomy and the number of supersymmetries are also slightly different.

Just as was the case in the previous studies, [12–17] it is convenient to employ the method of Borokhov and Gubser [26] to look for the non-supersymmetric linearized perturbation, corresponding to adding a small number of anti D2-branes, to the BPS background at $Q = 0$. It turns out that there is one subtlety which manifests itself in the presence of the D2, D4, and D6 charges which requires special attention. The issue stems from distinguishing between D2 charges generated by explicitly adding a D2 brane from the D2 charge induced by gradually increasing the NSNS B -field in the presence of a D4-brane inducing an effective D2-brane charge. The former changes the brane charge and the Page charge without changing b_∞ . The latter changes the brane charge and b_∞ but does not change the Page charge. Strictly speaking, from the point of view of classical supergravity which is only sensitive to $H_3 = dB_2$, the two procedures are indistinguishable. Yet, they are distinct in the full quantum interpretation and in the context of gauge/gravity duality.

The issue of varying a locally exact piece of NSNS 2-form with a period on some 2-cycle in the presence of a $p + 2$ brane wrapping that 2-cycle is a bit subtle when accounting for the charge of a p brane as was reviewed in appendix A of [5]. At the level of classical supergravity, one can imagine deforming the background by a) adding a locally exact 2-form to the B -field without adding any additional charge source, b) add a charge source without changing the B -field asymptotically, or c) perform a combination of the two.

Clearly, only two out of these three deformations are linearly independent at the level of classical gravity. If one does not distinguish backgrounds which differ only by a closed term in B_2 so that the two H_3 are indistinguishable, only one out of these three deformations would

appear to be physical. In the A_8 , however, b_∞ is a parameter specifying the background that must be kept account of. One must therefore be very explicit in making sure both of the linearly independent components of a), b), and c) are included in the space of deformations. This issue is closely related to the fact that we have D2, D4, and D6 charges, and that tuning the locally exact part of B affects not only the D2 brane charge but also the D4 brane charge, whereas we wish to adjust them independently.

In order to spell out this issue, we find it convenient to first study the linearized supergravity analysis for the anti D2-branes in flat space. This will turn out to also be a useful framework to review the formalism Borokhov and Gubser. After working out this simple exercise of identifying the anti D2-brane in flat space, it is straightforward to generalize the procedure to the warped A_8 case and to highlight the important features.

3.1 Linearized analysis for branes in flat space

Consider the truncated M-theory action

$$S = \int d^{11}x \sqrt{-\det(g_{11})} \left(R - \frac{1}{2} |F_4|^2 \right) \quad (3.1)$$

and consider a simple ansatz

$$\begin{aligned} ds^2 &= e^{-2z} \eta_{\mu\nu} dx^\mu dx^\nu + e^z (h^2 dr^2 + \ell^2 g^2 d\Omega_7^2) \\ C_3 &= e^{-3\tilde{z}} dx^0 \wedge dx^1 \wedge dx^2 \end{aligned} \quad (3.2)$$

where z , \tilde{z} , g , and h are the a priori independent fields. To allow for the possibility of finding non-BPS solutions, we are parametrizing the warp factor

$$H = e^{3z} \quad (3.3)$$

and the electric 3 form potential

$$\tilde{H}^{-1} = e^{-3\tilde{z}} \quad (3.4)$$

as independent variables. The effective action for the radial dependence of these fields takes the form

$$S_{eff} = \int dr g^7 h \left(42 \frac{1}{g^2} + 42 \frac{\ell^2}{h^2} \left(\frac{g'}{g} \right)^2 - \frac{9\ell^2}{2h^2} (z')^2 + \frac{9\ell^2}{2h^2} (\tilde{z}')^2 e^{6(z-\tilde{z})} \right) \quad (3.5)$$

The field h is non-dynamical and reflects the fact that it can be fixed to take on an arbitrary form by reparametrizing the radial variable. Let us choose⁴

$$\frac{1}{\ell} h dr = -g^7 d\tau \quad (3.6)$$

⁴We are treating τ as a dimensionless variable whereas r has the dimension of length.

so that the effective action becomes

$$S = \int d\tau \left(42g^{12} + 42 \left(\frac{g'}{g} \right)^2 - \frac{9}{2} (z')^2 + \frac{9}{2} (\tilde{z}')^2 e^{6(z-\tilde{z})} \right) \quad (3.7)$$

The solutions derived from this effective action are also subject to the zero energy condition

$$\left(42g^{12} - 42 \left(\frac{g'}{g} \right)^2 + \frac{9}{2} (z')^2 - \frac{9}{2} (\tilde{z}')^2 e^{6(z-\tilde{z})} \right) = 0 \quad (3.8)$$

from varying (3.5) with respect to h .

In [16], a trick is used to substitute

$$K = -(e^{-3\tilde{z}})' \quad (3.9)$$

and write the action in the form

$$S = \int d\tau \left(42g^{12} + 42 \left(\frac{g'}{g} \right)^2 - \frac{9}{2} (z')^2 + \frac{1}{2} K^2 e^{6z} \right) \quad (3.10)$$

and eliminate K algebraically. Such introduction of auxiliary variable K is useful for later purposes when we set up a superpotential to characterize the BPS equations and their small perturbations, but is not quite correct in the present form. The equation of motion derived from variation of \tilde{z} implies that K is constant, not zero.

One way to address this is to include a Lagrange multiplier field $q(\tau)$ to impose the constraint that $K = 3\tilde{z}'e^{-3\tilde{z}}$

$$S = \int d\tau \left(42g^{12} + 42 \left(\frac{g'}{g} \right)^2 - \frac{9}{2} (z')^2 + \frac{1}{2} K^2 e^{6z} + q(\tau)(K + (e^{-3\tilde{z}})') \right) \quad (3.11)$$

Then, integrating out q and then K will reproduce (3.7). If, instead, one integrates out \tilde{z} first, one infers that $q(t) = q = \text{constant}$. Further integrating out K takes the effective action to the form

$$S = \int d\tau \left(42g^{12} + 42 \left(\frac{g'}{g} \right)^2 - \frac{9}{2} (z')^2 - \frac{1}{2} q^2 e^{-6z} \right) \quad (3.12)$$

The parameter q enters as one of the variables controlling

$$-(e^{-3\tilde{z}})' = K = qe^{-6z} \quad (3.13)$$

and integrating this equation will give rise to one more integration constant, associated with the degrees of freedom \tilde{z} .

We can now proceed to analyze the BPS background and a first order deformation around it following the method of [26].

First, note that the effective action (3.12) can be written in the form

$$\int d\tau (T - U) \quad (3.14)$$

where

$$U = -\frac{1}{2}G^{ij}\frac{\partial W}{\partial\phi^i}\frac{\partial W}{\partial\phi^j} \quad (3.15)$$

$$T = \frac{1}{2}G_{ij}(\phi^i)'(\phi^j)' \quad (3.16)$$

for

$$\{\phi^1, \phi^2\} = \{g, z\} \quad (3.17)$$

$$G_{ij} = \begin{pmatrix} \frac{84}{g^2} & \\ & -9 \end{pmatrix}_{ij} \quad (3.18)$$

and

$$W = -14g^6 + qe^{-3z} \quad (3.19)$$

A BPS solution can be found by solving

$$\frac{d\phi^i}{d\tau} - G^{ij}\frac{\partial W}{\partial\phi^j} = 0 . \quad (3.20)$$

The solution is

$$\phi_0^1 = (6\tau)^{-1/6} \quad (3.21)$$

$$\phi_0^2 = \frac{1}{3}\log(q\tau + 1) \quad (3.22)$$

which translates to

$$\begin{aligned} e^{3z} &= 1 + \frac{q\ell^6}{6r^6} \\ g &= \frac{r}{\ell} \end{aligned} \quad (3.23)$$

under

$$r^6 = \ell^6(6\tau)^{-1} . \quad (3.24)$$

Here, ℓ is some generic length scale introduced to keep track of dimensions. In order to normalize q to the standard M-theory charge conventions, we see that we should set

$$\frac{q\ell^6}{6} = 32\pi^2(l_p^{11})^6 Q . \quad (3.25)$$

To study the solution to the equation of motion at first order, we expand

$$\phi^i = \phi_0^i + \phi_1^i \quad (3.26)$$

and derive the equation

$$\begin{aligned} \frac{d\xi_i}{d\tau} &= -\xi_j N^j{}_i, & N^i{}_j &= \frac{\partial}{\partial \phi^j} G^{ik} \frac{\partial W}{\partial \phi^k} \\ \frac{d\phi_1^i}{d\tau} &= N^i{}_j \phi_1^j + G^{ij} \xi_j, \end{aligned} \quad (3.27)$$

satisfied by the ϕ_1^i , as well as the set of auxiliary field ξ_i . In the Borokhov-Gubser formalism, n sets of second order differential equations for the ϕ_1^i fields are reformulated as $2n$ sets of first order differential equations for the ϕ_i 's and the ξ_i 's.

These equations are solved while treating q also as a first order perturbation. However, since q is not one of the parameters which affects the fields ξ_i and ϕ_1^i at the linear order, the term linear in q needs to be included as part of the zero-th order solution. We are therefore considering an expansion around the zeroth order solution

$$\begin{aligned} \phi_0^1 &= (6\tau)^{-1/6} \\ \phi_0^2 &= \frac{1}{3}q\tau \end{aligned} \quad (3.28)$$

and these are the background functions which go into $N^j{}_i$ and G^{ij} in (3.27)

These equations are solved in the following order.

1. ξ_2 is dictated by

$$\frac{d\xi_2}{d\tau} = 0 \quad (3.29)$$

and is solved by

$$\xi_2 = X_2 \quad (3.30)$$

2. The equation for ϕ_1^2 is given by

$$\frac{d\phi_1^2}{d\tau} = -\frac{1}{9}X_2 \quad (3.31)$$

and is solved by

$$\phi_1^2 = -\frac{1}{9}X_2\tau + Y_2 \quad (3.32)$$

3. The equation for ξ_1 is given by

$$\frac{d\xi_1}{d\tau} = \frac{7}{6\tau}\xi_1 \quad (3.33)$$

and is solved by

$$\xi_1 = X_1(6\tau)^{7/6} \quad (3.34)$$

4. Finally, ϕ_1^1 satisfies

$$\frac{d\phi_1^1}{d\tau} = -\frac{7}{6\tau}\phi_1^1 + \frac{1}{84}X_1(6\tau)^{5/6} \quad (3.35)$$

and is solved by

$$\phi_1^1 = \frac{X_1}{1521}(6\tau)^{11/6} + Y_1(6\tau)^{-7/6} \quad (3.36)$$

The zero energy condition (3.8) for these fields is given by

$$\xi_i \frac{d\phi_0^i}{d\tau} = -X_1 = 0 \quad (3.37)$$

The general linearized solution we found can be summarized as

$$g = (6\tau)^{-1/6} + (Y_1(6\tau)^{-7/6}) \quad (3.38)$$

$$H = e^{3z} = 1 + \left(q\tau - \frac{1}{3}X_2\tau + Y_2 \right) \quad (3.39)$$

$$\tilde{H}^{-1} = e^{-3\tilde{z}} = -q\tau + \delta \quad (3.40)$$

where δ is the integration constant we inherit from integrating (3.13). In terms of the more conventional radial variable

$$r^6 = \ell^6(6\tau)^{-1} \quad (3.41)$$

these solutions take the form

$$g = r + \left(Y_1 \frac{r^7}{\ell^7} \right) \quad (3.42)$$

$$H = e^{3z} = 1 + \left(\left(q - \frac{1}{3}X_2 \right) \frac{\ell^6}{r^6} + Y_2 \right) \quad (3.43)$$

$$\tilde{H}^{-1} = e^{-3\tilde{z}} = -q \frac{\ell^6}{r^6} + \delta \quad (3.44)$$

Special cases of these expressions correspond to familiar solutions. It is convenient to set $Y_1 = Y_2$ so that the geometry asymptotes to flat space in the canonical metric for large r . The parameter δ is pure gauge and so it can be set to zero without any harm. Further setting $X_2 = 0$ will give rise to the linearized form of the BPS solution identified earlier (3.23) for $q > 0$ [27] with q normalized according to (3.25). Choosing $X_2 = 6q$ and taking $q < 0$ corresponds to what we would identify as the anti M2-brane. Other generic values of X_2 appear to correspond to the solution found in (5.12) of [28] with $\alpha = 0$, $D = 11$, $p = 2$. One can also continue to compute higher order corrections to ϕ_2^i , ϕ_3^i , etc, and reproduce the entire non-linear solutions [27, 28].

3.2 Back reaction of anti-branes in warped A_8 geometry

We will now face the beast, and address the problem of computing the gravitational back reaction of anti D2-branes for the warped A_8 background working to first order around the $Q = 0$ background.

The supergravity equations of motion are inferred from the full bosonic M-theory action

$$S = \frac{1}{2\kappa^2} \int d^{11}x \sqrt{-\det(g_{11})} \left(R - \frac{1}{2} |F_4|^2 \right) - \frac{1}{6} \int A_3 \wedge F_4 \wedge F_4 \quad (3.45)$$

The ansatz we consider is a generalization of what we reviewed in section 2.1 where the warp factor e^{3z} and the electric components of the 4-form $e^{3\tilde{z}}$ are allowed to vary independently. Explicitly, they are given by

$$\begin{aligned} ds_{11}^2 &= e^{-2z} \eta_{\mu\nu} dx^\mu dx^\nu + e^z (ds_8^2) \\ ds_8^2 &= h^2 dr^2 + \ell^2 \left(a^2 (D\mu^i)^2 + b^2 \sigma^2 + c^2 d\Omega_4^2 \right) \\ C_3 &= e^{-3\tilde{z}} dx^0 \wedge dx^1 \wedge dx^2 + m (v_1 \sigma \wedge X_{(2)} + v_2 \sigma \wedge Y_{(2)} + v_3 Y_{(3)}) + w d\sigma \wedge d\varphi \end{aligned} \quad (3.46)$$

We are looking for a solution which depends on a single radial variable r . When this ansatz is substituted into the action (3.45), we can derive a lengthy effective action

$$\begin{aligned} S &= \int dr \left\{ \frac{a^2 b c^4}{h} \left[2(\alpha')^2 + 12(\gamma')^2 + 4\alpha'\beta' + 16\alpha'\gamma' + 8\beta'\gamma' - \frac{9}{2}(z')^2 + \frac{9}{2}(\tilde{z}')^2 e^{6(z-\tilde{z})} \right] \right. \\ &\quad - \frac{1}{2} \frac{h}{a^2} b (-4a^2 c^4 - 24a^4 c^2 + 4a^6 + b^2 c^4 + 2a^4 b^2) \\ &\quad - \frac{m^2}{\ell^6} \left[\frac{1}{2he^{3z}} (e^{4\gamma-\beta-2\alpha}(v'_1)^2 + 2e^{2\alpha-\beta}(v'_2)^2 + 4e^\beta(v'_3)^2) \right. \\ &\quad + he^{-3z} (2e^{-\beta}(v_1+v_2)^2 + e^{\beta-2\alpha}(v_2-v_1+2v_3)^2 + 2e^{2\alpha+\beta-4\gamma}(2v_3-v_2)^2) \\ &\quad \left. \left. + (3\tilde{z}'e^{-3\tilde{z}})(4v_3(v_1+v_2) + v_2(v_2-2v_1)) \right] \right\} \end{aligned} \quad (3.47)$$

where

$$a = e^\alpha, \quad b = e^\beta, \quad c = e^\gamma. \quad (3.48)$$

All of these fields $a, b, c, h, v_1, v_2, v_3, z$, and \tilde{z} can be viewed as being dimensionless. r and ℓ have dimension of length.

We can also use the reparametrization invariance of the radial variable to put h into a convenient form. We find it best to introduce the dimensionless radial variable τ using

$$h dr = -\ell a^2 b c^4 d\tau. \quad (3.49)$$

This eliminates h as an effective degree of freedom aside from imposing the zero energy condition.

In order to apply the procedure of Borokhov and Gubser, we would like to organize this action into a form where

$$S = - \int d\tau (T - U) \quad (3.50)$$

where the potential U can be expressed in terms of some superpotential. This is accomplished by the procedure of integrating out $K = (e^{-3\tilde{z}})'$ at the expense of introducing an auxiliary parameter q as we did in (3.9).

These manipulations will bring the effective action into the form (3.50), with

$$T = 2(\alpha')^2 + 12(\gamma')^2 + 4\alpha'\beta' + 16\alpha'\gamma' + 8\beta'\gamma' - \frac{9}{2}(z')^2 - \frac{m^2}{\ell^6} \frac{1}{2a^2bc^4e^{3z}} (e^{4\gamma-\beta-2\alpha}(v'_1)^2 + 2e^{2\alpha-\beta}(v'_2)^2 + 4e^\beta(v'_3)^2) \quad (3.51)$$

$$U = \frac{1}{2}b^2c^4(-4a^2c^4 - 24a^4c^2 + 4a^6 + b^2c^4 + 2a^4b^2) + \frac{m^2}{\ell^6} \frac{a^2bc^4}{e^{3z}} (2e^{-\beta}(v_1 + v_2)^2 + e^{\beta-2\alpha}(v_2 - v_1 + 2v_3)^2 + 2e^{2\alpha+\beta-4\gamma}(2v_3 - v_2)^2) + \frac{e^{-6z}}{2} \left(\frac{m^2}{\ell^6} (4v_3(v_1 + v_2) + v_2(v_2 - 2v_1)) + q \right)^2 \quad (3.52)$$

for

$$W = -bc^2(4a^3 - 2a^2b + 4ac^2 + bc^2) + e^{-3z} \left(\frac{m^2}{\ell^6} (4v_3(v_1 + v_2) + v_2(v_2 - 2v_1)) + q \right) \quad (3.53)$$

To identify the background solution, we set up the BPS equation (3.20) for

$$\{\phi^1, \phi^2, \phi^3, \phi^4, \phi^5, \phi^6, \phi^7\} = \{\alpha, \beta, \gamma, z, v_1, v_2, v_3\} . \quad (3.54)$$

with \tilde{z} is determined by the auxiliary condition

$$3\tilde{z}'e^{6z-3\tilde{z}} = q + \frac{m^2}{\ell^6} (4v_3(v_1 + v_2) + v_2(v_2 - 2v_1)) . \quad (3.55)$$

One can immediately confirm, for example, that flat space in eleven dimensions,

$$a = b = c = (3\tau)^{-1/6} \quad v_i = 0 , \quad (3.56)$$

is a solution, which can be presented in the standard form by parametrizing

$$3\tau = \left(\frac{2\ell}{r} \right)^6 . \quad (3.57)$$

Similarly, the BPS A_8 solution of section 2.1 can be shown to solve the BG equations. The BPS equation (3.20) is invariant under reparametrization of coordinates as long as one suitably transforms

$$G^{ij} \rightarrow \frac{d\tau}{dr} G^{ij} \quad (3.58)$$

and one can confirm that the A_8 solution is indeed a solution of (3.20) in the r coordinates. The r and τ coordinates are related by

$$\tau = - \int \frac{dr}{\ell} \frac{h}{a^2 b c^4} = \frac{\ell^3}{3(r-\ell)^3} - \frac{\ell^2}{4(r-\ell)^2} + \frac{3\ell}{16(r-\ell)} + \frac{\ell}{16(r+3\ell)} + \frac{1}{16} \log \frac{r-\ell}{r+3\ell}. \quad (3.59)$$

In the dimensionless τ coordinates, the kinetic term the metric G_{ij} take on a relatively simple form.

Expanding in inverse power of τ is equivalent to expanding near the tip $r = \ell$. The warp factor (2.10) for the warped A_8/Z_k background can be written, as an expansion in τ^{-1} , as

$$e^{3z} = q\tau + \left(\frac{m^2}{\ell^6} \frac{23}{5 \times 2^{12}} - q\mathcal{O}(\tau^0) \right) + \mathcal{O}(\tau^{-1/3}). \quad (3.60)$$

Comparing to the form of (2.10), we find that

$$q\ell^6 = \frac{3}{2}\pi^2(l_p^{11})^6 kQ \quad (3.61)$$

where q is the parameter appearing in the superpotential and Q is the D2 charge normalized such that a single D2-brane has charge one.

We will take this solution at $q = 0$ as our background solution ϕ_0^i and expand

$$\phi^i = \phi_0^i + \phi_1^i \quad (3.62)$$

and attempt to find the ξ_i and ϕ_1^i which describe the back reaction of an anti D2-brane to first order. We also allow q to shift at the same order as ξ_i and ϕ_1^i .

To proceed further, we need to take a closer look at the structure of the matrix N^j_i entering the Borokhov-Gubser equation (3.27). To this end, it is convenient to group the 7 fields ϕ_1^i into subgroups which we might refer to as

$$\begin{aligned} \phi_{geom} &= \{\phi_1^1, \phi_1^2, \phi_1^3\} = \{\alpha, \beta, \gamma\} \\ \phi_z &= \{\phi_1^4\} = \{z\} \\ \phi_{flux} &= \{\phi_1^5, \phi_1^6, \phi_1^7\} = \{v_1, v_2, v_3\} \end{aligned} \quad (3.63)$$

In this classification, N^j_i has a block structure which can be summarized by

$$N^j_i = \left(\begin{array}{c|c|c} N_{geom} & & \\ \hline & N_z & N_{zf} \\ \hline N_{gf} & & N_{flux} \end{array} \right)^j_i \quad (3.64)$$

There are various blocks which have vanishing entries, which suggests a strategy for the order of solving the Borokhov-Gubser equations. Specifically

1. We first solve for ξ_4 which is decoupled and can be solved in closed form,
2. then we solve for $\xi_{flux} = \xi_{5,6,7}$ which takes ξ_4 as a source but is otherwise decoupled,
3. then, we solve for $\xi_{geom} = \xi_{1,2,3}$ which takes other ξ 's as sources but are otherwise closed,
4. then, $\phi^{geom} = \phi_1^{1,2,3}$ form a closed set of equations taking ξ 's as sources,
5. then, $\phi^{flux} = \phi_1^{5,6,7}$ can be solved taking ϕ^{geom} and ξ 's as sources,
6. and finally, ϕ_1^4 can be computed.

Although these steps are straightforward in principle, the fact that the intermediate step involves diagonalizing a coupled system of three first order differential equation makes this exercise somewhat formidable to execute in complete form. This is in contrast to earlier instances such [12,13,16] where the mixing only involved two fields which were significantly easier to diagonalize. Fortunately, this analysis is still tractable if we restrict the scope of our study to explore the asymptotic behavior near $r = \ell$ or large τ . It will turn out that this suffices for the conclusion we are after. Let us now proceed to describe this analysis in more detail. It should be stressed that aside from logistical challenges, nothing prevents us from attempting to explore the solution to the full system of equations numerically.

Let us follow this procedure step by step.

1. The first step of solving for ξ_4

$$\frac{d\xi_4}{d\tau} = -N^4 {}_4\xi_4 = e^{-3z} \left(\frac{m^2}{\ell^6} (4v_3(v_1 + v_2) + v_2(v_2 - 2v_1)) + q \right) \xi_4 \quad (3.65)$$

which, using (3.55) can be written to linear order in q and ξ_4 as

$$\frac{d\xi_4}{d\tau} = 3z' e^{3z} \xi_4 \quad (3.66)$$

can be solved by

$$\xi_4 = X_4 e^{3z} \sim X_4 \frac{m^2}{\ell^6} \frac{23}{5 \times 2^{12}} + \mathcal{O}(\tau^{-1/3}) \quad (3.67)$$

where X_4 is the integration constant for the ξ_4 equation, and we used (3.60) in the last step.

2. Now we are ready to consider the equations for ξ_{567} which can be written as

$$\begin{pmatrix} \xi'_5 \\ \xi'_6 \\ \xi'_7 \end{pmatrix} = \begin{pmatrix} 0 & -e^{2\beta+4\gamma} & e^{2\alpha+4\gamma} \\ -2e^{4\alpha+2\beta} & e^{2\beta+4\gamma} & e^{2\alpha+4\gamma} \\ 4e^{4\alpha+2\beta} & 2e^{2\beta+4\gamma} & 0 \end{pmatrix} \begin{pmatrix} \xi_5 \\ \xi_6 \\ \xi_7 \end{pmatrix} + \frac{2m^2}{3\ell^6} X_4 \begin{pmatrix} v_2 - 2v_3 \\ v_1 - v_2 - 2v_3 \\ -2(v_1 + v_2) \end{pmatrix} \quad (3.68)$$

Now, this is a rather cumbersome equation to solve in closed form, but in the large τ $r \rightarrow \ell$ limit, reduces to

$$\begin{pmatrix} \xi'_5 \\ \xi'_6 \\ \xi'_7 \end{pmatrix} = \frac{1}{3\tau} \begin{pmatrix} 0 & -1 & 1 \\ -2 & 1 & 1 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} \xi_5 \\ \xi_6 \\ \xi_7 \end{pmatrix} + \frac{m^2}{\ell^6} \frac{X_4}{16(3\tau)^{\frac{2}{3}}} \begin{pmatrix} \frac{1}{3} \\ -\frac{1}{2} \\ \frac{1}{6} \end{pmatrix} \quad (3.69)$$

These equations can be solved, introducing integration constants $X_{5,6,7}$

$$\begin{aligned} \xi_5 &= \frac{1}{2}(X_6 - X_7)\tau^{2/3} - \frac{1}{2}X_5\tau^{-1} - \frac{3^{1/3}m^2}{48\ell^6}X_4\tau^{1/3} \\ \xi_6 &= X_7\tau^{2/3} - \frac{1}{2}X_5\tau^{-1} + \frac{3^{1/3}m^2}{32\ell^6}X_4\tau^{1/3} \\ \xi_7 &= X_6\tau^{2/3} + X_5\tau^{-1} - \frac{1}{32 \times 3^{2/3}}\frac{m^2}{\ell^6}X_4\tau^{1/3} \end{aligned} \quad (3.70)$$

3. Now we feed these $\xi_{4,5,6,7}$ into the $\xi_{1,2,3}$ equations and follow the same steps. Below we only indicate the terms which are singular in the large τ limit.

$$\begin{aligned} \xi_1 &= \frac{1}{2}X_2\tau \\ \xi_2 &= \frac{1}{4}X_2\tau - X_3\tau^{1/3} \\ \xi_3 &= X_2\tau + X_3\tau^{1/3} \end{aligned} \quad (3.71)$$

4. Next, we compute $\phi_1^{1,2,3}$.

$$\begin{aligned} \phi_1^1 &= \frac{m^2}{\ell^6} \frac{1}{32,768} X_4\tau^{2/3} - \frac{41}{3,072 \times 3^{2/3}} X_6\tau - \frac{1}{384 \times 3^{2/3}} X_7\tau \\ &\quad - \frac{5}{2}(0X_3 + Y_1)\tau^{4/3} + \frac{X_2}{144}\tau^2 \\ \phi_1^2 &= \frac{m^2}{\ell^6} \frac{73}{98,304 \times 3^{1/3}} X_4\tau^{2/3} + \frac{(-8X_6 + 25X_7)}{768 \times 3^{2/3}} \tau \\ &\quad + \frac{1}{10}(3X_3 + 10Y_1)\tau^{4/3} + \frac{X_2}{144}\tau^2 \\ \phi_1^3 &= -\frac{m^2}{\ell^6} \frac{25}{98,304 \times 3^{1/3}} X_4\tau^{2/3} + \frac{(31X_6 - 26X_7)}{3,072 \times 3^{2/3}} \tau \\ &\quad + \frac{1}{40}(-3X_3 + 40Y_1)\tau^{4/3} + \frac{X_2}{144}\tau^2 \end{aligned} \quad (3.72)$$

We have included the X_3 term in ϕ_1^1 whose coefficient is accidentally zero. In generic linear combinations of $\phi_1^{1,2,3}$, X_3 would appear in that order.

5. Now, computing the $\phi_1^{5,6,7}$ along similar lines, we find

$$\begin{aligned}
\phi_1^5 &= \frac{23}{983,040 \times 3^{2/3}} \frac{m^2}{\ell^6} X_4 \tau^{1/3} - \frac{23}{163,840} (X_6 - X_7) \tau^{2/3} \\
&\quad - \frac{3^{1/3}}{640} (X_3 - 55Y_1) \tau^{2/3} - 2Y_5 \tau + \frac{X_2}{2,304 \times 3^{2/3}} \tau^{4/3} \\
\phi_1^6 &= -\frac{23}{1,310,720 \times 3^{2/3}} \frac{m^2}{\ell^6} X_4 \tau^{1/3} - \frac{23}{163,840} (0X_6 + X_7) \tau^{2/3} \\
&\quad - \frac{(27X_3 - 310Y_1)}{2,560 \times 3^{2/3}} \tau^{2/3} - Y_5 \tau - \frac{X_2}{3,072 \times 3^{2/3}} \tau^{4/3} \\
\phi_1^7 &= \frac{23}{7,864,320 \times 3^{2/3}} \frac{m^2}{\ell^6} X_4 \tau^{1/3} \\
&\quad + \frac{(704 \times 3^{1/3} X_3 - 21,120 \times 3^{1/3} Y_1 - 23X_6 + 0X_7)}{327,680} \tau^{2/3} \\
&\quad + Y_5 \tau + \frac{X_2}{18,432 \times 3^{2/3}} \tau^{4/3}
\end{aligned} \tag{3.73}$$

The coefficients Y_5 , Y_6 , and Y_7 can be viewed as parameterizing the magnitude of self-dual 4-forms of which only one linear combination corresponds to the square normalizable 4-forms.

6. Finally,

$$\begin{aligned}
\phi_1^4 &= -\left(\frac{49 \times 3^{2/3}}{1,656} X_3 + \frac{23}{184,320} \frac{m^2}{\ell^6} X_4 - \frac{5 \times 3^{1/3}}{4,608} (X_6 - 2X_7) - \frac{35 \times 3^{2/3}}{828} Y_1 \right) \tau \\
&\quad + Y_4 + 0Y_1 \tau^{4/3} - \frac{35}{9,936 \times 3^{1/3}} X_2 \tau^{5/3} + \frac{5 \times 2^{12} \ell^6}{69m^2} q \tau
\end{aligned} \tag{3.74}$$

At this stage, we have enumerated the terms in ϕ_1^i which are singular in the $\tau \rightarrow \infty$ limit, corresponding to probing the near core region $r \sim \ell$. These expressions should be viewed as representing, for each X_i and Y_i , the leading singular τ dependence in the large τ limit. Strictly speaking, these solutions depend on all 14 X_i 's and Y_i 's, except that only those which are singular are presented. We have also included the dependence on q which is also to be treated at the same order.

Out of this 15 dimensional space of linearized solutions, we wish to identify the particular deformation corresponding to adding a specific amount of anti D2-branes. We will attempt to determine the unique linear combination based on 1) the zero energy condition, 2) consideration of the effect of the deformation on the Page charge 3) the expected forces on D2-brane probe, and 4) the requirement to keep the solution regular near $r = \ell$.

As have been the case in many of the earlier works on related systems, it will turn out that satisfying all of the 4 requirements appears to be impossible.

1. Let us first consider the zero-energy condition which in the $r \approx \ell$ limit simplifies for these linear solutions to

$$0 = \xi_i \frac{d\phi_0^i}{d\tau} = -\frac{7}{24} X_2 \quad (3.75)$$

Thus we need $X_2 = 0$ to first order. Next, let us consider the effects of deformations X_i 's and Y_i 's on D2, D4, and D6 Page charges. It turns out that all of these do not affect any of the D2, D4, and D6 Page charges. The only parameter which affects the D2 Page charge is the parameter q . So, the linear deformation interpretable as adding anti D2-brane must shift q .

2. Next, we consider the force on D2-brane probe which is expected for adding an anti D2-brane. This can be computed using the standard DBI analysis giving rise to, using (3.55)

$$\begin{aligned} F &= F^{DBI} + F^{WZ} \\ &= -3z'e^{-3z} + 3\bar{z}'e^{-3\bar{z}} \end{aligned} \quad (3.76)$$

which for the first order deformation simplifies to

$$F = \frac{1}{H(\ell)^2} \frac{23}{184,320} \frac{m^2}{l^6} X_4 \quad (3.77)$$

3. Next, from the form of (3.61) and (3.77), we also infer that for $-Q$ anti D2-branes, we should scale

$$X_4 = -3 \left(\frac{5 \times 2^{12}}{23 \frac{m^2}{l^6}} \right)^2 q = -2 \left(\frac{15 g_s^2 k^2}{46} \right)^2 \frac{k^3 Q}{M^4} \quad (3.78)$$

4. Next, we examine the divergences in the ϕ_1^i 's in the core region. Looking at terms diverging as $\tau^{4/3}$ in $\phi_1^{1,2,3}$, we infer that Y_1 and X_3 should be set to zero. This leaves terms diverging as τ in $\phi_1^{1,2,3}$. From this, we see that X_6 and X_7 should also be set to zero to prevent singularities in the core region. So far, from looking at $\phi_1^{1,2,3}$ alone, we have set

$$\{X_3, X_6, X_7, Y_1\} \quad (3.79)$$

to zero, in addition to X_2 which had to vanish because of the zero energy condition. We will hold off on addressing X_4 for now since that mode plays a special role in coupling to the D2-brane probe.

Just from these constraints, we see that $\phi_1^{4,5,6}$ are also severely constrained. Aside from X_4 , the only remaining integration constant is Y_5 . Y_5 appears to be an interesting mode which we will further discuss elsewhere. It corresponds to deformation by non-normalizable self-dual 4 form, as can be seen as arising as a Y -deformation, which is supersymmetry preserving.

The other constants

$$\{X_1, X_5, Y_2, Y_3, Y_6, Y_7\} \quad (3.80)$$

do not appear to induce divergent terms in the core region.

The ultimate question whose answer we seek is whether one can deform the solution to incorporate the back reaction of anti D2-branes while preventing additional singularities from appearing. It appears that the answer, as was the case in many earlier attempts in related systems, is “no.” In order to capture the tension of the anti D2-brane in the warp factor, we have to turn on X_4 . That X_4 turns on $\tau^{2/3}$ singularities in $\phi_1^{1,2,3}$ and $\tau^{1/3}$ singularities in $\phi_1^{5,6,7}$, and there are no remaining adjustable integration constants one can turn on to cancel these singularities without generating other singularities elsewhere.

This conclusion is not extremely surprising. The narrative of how the singularities in fluxes and the back reaction of the anti D2-brane tension imposes conflicting constraints is identical to that which was found in earlier analysis of similar constructions [12–17]. One novel feature in our analysis is the explicit absence of any smearing of the anti-brane sources. But this does not appear to have much effect on the conclusion.

This however raises a question concerning the fate of the non-supersymmetric ground state anticipated to encapsulate the features illustrated in figure 2. In the consideration of the meta-stable vacua, it was always possible for the state to destabilize under some repulsive effect generated by the breaking of supersymmetry. In the case of the Chern-Simons theories under consideration, however, one expects there to be a competing restorative component to balance the dynamically generated repulsive force. Also, unlike in the Klebanov-Strassler case, there are no alternative BPS supergravity solution for a given N , l , k , and b_∞ where $Q < 0$. So there are no supersymmetric vacua for the non-supersymmetric state to decay into.

In the discussion section, we will offer our speculation concerning the fate of $Q < 0$ theories from the perspective of the gravity dual.

4 Discussions

In the earlier sections, we formulated and analyzed the construction of supergravity solutions corresponding to a warped A_8 geometry parametrized by N , l , k , and b_∞ . In the analysis, it became clear that there is a parameter,

$$Q = N - \frac{l(l-k)}{2k} - \frac{k}{24} \quad (4.1)$$

which if positive, gives rise to a sensible warped supergravity solution with an asymptotic anti-de-Sitter region in the core. At $Q = 0$, we also found that there is a sensible supergravity solution describing the geometry in the core region. The question was whether one could explore the backgrounds for $Q < 0$. We examined this as an exercise in incorporating the gravitational back reaction of anti D2-branes added to the $Q = 0$ background and studied the effect of this operation at linear order in shift in Q . What we found is that sensible perturbation respecting regularity in the core region while accounting for the physical features of the anti D2-branes could not be found.

Implicit in this thinking is the notion that starting from $Q = 0$ solution, making Q positive corresponds to adding a D2-brane and making Q negative corresponds to adding an anti D2-brane. This is the perfectly sensible way in which things work in familiar context such as flat space which we reviewed in section 3.1.

One potential fallacy is the assumption that the switch from branes to anti-branes should happen at $Q = 0$ also for the A_8/Z_k background. A hint that something might be tricky here stems from the curvature correction term $-k/24$ in the expression for Q . Strictly speaking, a correction of this form should be considered as part of α' correction to supergravity since we assume we are working in the strong 't Hooft coupling limit which instructs us to take $Q_2^{Maxwell}$ to be large which amounts to considering $x = N/k$ and $y = l/k$ to be large. The only reason we have to take the $-k/24$ term seriously was the work of [6] which, on the field theory side, computed the free energy precisely for arbitrary 't Hooft coupling, not just its strong coupling asymptotics. This means that even in the A_8 background with no D2 or D4 branes added, i.e. with $N = l = 0$, the geometry has some D2 brane charge due to curvature effects, and it contributes negatively.

Generally, in string theory, singular BPS geometries are considered physically allowed if the object sourcing them exists in the theory. For example, large curvature singularity near the fundamental string solution is considered an acceptable singularity because a fundamental string is part of string theory. Moreover, negatively charged negative tension objects do not appear in classical gravity, but the ones which arise from curvature corrections of orbifolds and orientifolds [29, 30] are exceptions to this rule. A situation similar to this was part of the repulson/enhancon construction [31].

Let us for a moment take the point of view that the class of solutions we found for $Q > 0$ can be extrapolated by changing Q not down to $Q = 0$, but rather down to $Q = -k/24$. The negative Q solution exhibits a repulson type singularity were a generic massive object will feel a repulsive force [32–34]. Some of the features of the repulson dynamics were discussed in [9] but were not taken too seriously at the time because it was believed that the repulson geometry itself should not be taken seriously. It is the fact that the curvature correction

$Q = -k/24$ appears also in the field theory analysis which offers a renewed motivation to take the repulson solution seriously, at least for $Q > -k/24$.

It is natural to wonder if this repulson singularity is resolved by the enhancon mechanism [31]. As far as we could tell, this is not the case. The only BPS probe we are able to find is the D2-brane. Looking at the kinetic term of the radial motion of D2-brane in this background, we did not find any locus interpretable as the enhancon radius.

What we do find, as was reported originally in [9], is that an anti D2-brane probe feels a repulsive force near the core but is stabilized to sit a finite radius. It is straightforward to compute the potential experienced by the anti-D2-brane probe. It is simply

$$V = 2T_2 H^{-1}(u) \quad (4.2)$$

where $u = r/\ell - 1 = 2U/g_{YM}^2$, and so is stabilized where

$$F(u) = V'(u) = 0 . \quad (4.3)$$

The potential for $Q = 0$ and small negative Q is illustrated in figure 3. A little computation shows that as a function of

$$\epsilon = -\frac{kQ}{M^2} \quad (4.4)$$

the stabilization point scales as

$$u \approx \epsilon^{1/4} \quad (4.5)$$

for small ϵ . This was also noted in [9].

If we take seriously the idea that it is $Q = -k/24$ and not $Q = 0$ at which we switch from subtracting branes to adding anti-branes as we decrease Q , the narrative of the evolution of the supergravity solution changes.

Let us take as given that we are always working with $Q \gg k$ so that the supergravity description is good, and $Q \ll M^2/k$ so that the departure away from BPS solution can be considered parametrically small. This of course implies that $M \gg k$. As we go from Q positive to Q negative in this parametrization, Q negative and small implies we have roughly n anti D2-branes which we have added to the $Q = -k/24$ solution for $1 \ll n/k \ll M$.

Out of these n anti D2-branes, imagine adding them one by one to the $Q = -k/24$ background. The first one will feel the potential (4.2) and settle at

$$u \approx \epsilon^{1/4} \approx \sqrt{\frac{k}{M}} \quad (4.6)$$

where we are using the fact that

$$Q = -\frac{k}{24} . \quad (4.7)$$

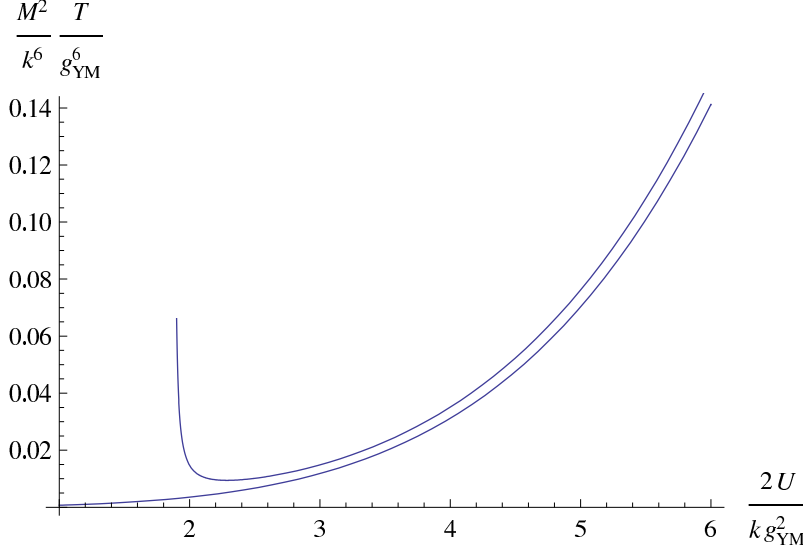


Figure 3: The potential experienced by an anti D2-brane in the $Q < 0$ background inferred from the DBI action. This plot includes the extreme case $\epsilon = 0$ and a weakly repulsive case $\epsilon = 0.01$.

For the purpose of making order of magnitude estimates, we have dropped the factor of $1/24$.

As more anti D2-branes are added, each will stabilize at roughly the same radius, as anti D2-branes do not sense each other's presence, and we are neglecting the back reactions of these probe anti D2's for the time being. It is natural to imagine these anti D2-branes forming a shell at the same radius as (4.6).

As even more anti D2's are added until we reach the total number n , one should not expect to get away with treating the anti D2's as a probe. The configuration one might expect to find is that of the $Q = -k/24$ geometry at the core, surrounded by a clump of anti D2-branes which back react to build the full geometry. It should be emphasized that these additional anti D2-branes do not make the repulson singularity any stronger. The strength of the repulson always corresponds to $Q = -k/24$, and the geometry is accompanied by a cloud of anti D2-branes which floats in some distribution, balancing the repulsion from the repulson and the stabilization due to the background flux parametrized by M . It seems natural to imagine that all of the n anti D2-branes stabilize at a radius of the order (4.6).

If this scenario is correct, one expects to find the supergravity solution, along the lines of what we found using the linearized perturbation around $Q = 0$, but pushing Q to be negative; i.e., we assume that the solution is valid for the radius outside the cloud of anti D2-branes. That some modes develop a singularity near the core is no longer a problem because once one hits the radius where the cloud of anti D2-branes is present, one is expected to cross over into a different behavior of the gravitational back reaction. In a sense, the cloud of anti

D2 branes shields the singularity implied by the perturbative analysis of section 3.

In order to extract meaningful physical quantities characterizing the dynamics of the $Q < 0$ phase of this theory, however, one must first come to grips with understanding how the anti D2-branes distribute themselves in the region characterized by the radius (4.6). Everything is happening at this very small length-scale and it appears to be beyond the scope of supergravity to settle this issue unambiguously. One opportunistic scenario is that the anti-D2-branes form a spherical shell of uniform density, and that one can construct a full back reacted solution by joining the $Q = -k/24$ solution on the inside and some generic $Q < 0$ solution on the outside with the suitable matching condition at some appropriate radius where a static solution can be shown to exist. It would be an interesting exercise to see if such a solution can be constructed.

Regardless of this issue, our proposal is that there exists a non-supersymmetric clump of anti D2-matter, stabilized by balancing a repulsive force from curvature correction and an attractive force of the background flux. This is a novel configuration of these objects in string theory and may be relevant to characterizing the state of other non-supersymmetric constructions.

Another consequence of this picture is the realization that the singularities encountered in the perturbative analysis (section 3) are a priori permissible because they can be regularized by the α' corrections. One should add that it is actually a bit of an over simplification. One can imagine that some of these divergences can get regularized by the α' effects, but we do not know if this is true of all singularities, nor how. In other words, in assessing the field theory observables such as the expectation values of some operators in the holographic language, one would be interested in finding which normalizable modes are activated in such a way that they are consistent with the boundary condition in the core region. That boundary condition is precisely the information encoded in the structure of the anti D2-brane clump in the core region as well as the presumed sub-stringy physics regularizing the repulsion. Understanding these issues brings the subject into the treacherous terrain of the study of stable non-BPS configurations in string theory and supergravity [35–40]. All of the quantitatively interesting information is encoded in the stringy dynamics, and appears to be beyond the scope of a simple space-time effective field theory analysis.

It should be emphasized, nonetheless, that the estimate of the mass gap (2.30) for $Q = 0$ is a reliable prediction of the dual gravity description. The scenario outlined above suggests that for sufficiently small ϵ , the scale of the gap will also make a small change, but we are unable to infer the precise scaling without making assumptions.

One may hope to make further progress on the field theory side. If the field theory side

can access information at all orders in the 't Hooft coupling, it will offer powerful insights into this phenomenon. Unfortunately, the technique employed in [6] is not applicable for probing $Q < 0$ since it relies on superconformal invariance as one of the key assumptions. By going to $Q < 0$, we are no longer able to rely on that feature.

Perhaps the most immediate task at hand is to explore the physics of the anti-brane clump in a more controlled setting. In this paper, we considered the regime $k \ll Q \ll M^2$ which forced M , the parameter controlling the attractive forces involved in stabilizing the anti D2-branes, to be large. This causes the size of the anti-brane clump to be small. It would be interesting to see if somehow one could make the strength M of the attractive force small so as to make the size of the clump large. In working with the supergravity dual of a decoupled field theory system, it was a requirement that M^2/k be large in order to ensure that the 't Hooft coupling is large. We can relax this requirement if we are going to study this issue as a brane dynamic issue in the A_8 background without taking the traditional $\alpha' \rightarrow 0$ limit where we “drop the 1” in the warp factor (2.10). With the “1” included, we can let M get close to the critical value $M^2 \sim -2Qk$ or $\epsilon = kQ/M^2 \sim 1/2$ for $Q \sim -k/24$ and still have asymptotically locally conical geometry. Indeed, for $M^2 \sim -2kQ$, one can show that the anti D2-brane stabilization radius

$$r_* \sim -\frac{16kQ}{M^2 + 2kQ} \sim \frac{\epsilon}{1 - 2\epsilon} \quad (4.8)$$

can get arbitrarily large as $\epsilon \rightarrow 1/2$. However, the curvature of the potential at the minimum

$$V''(r_*) \approx \frac{TQ}{g_s^4 k^5} (1 - 2\epsilon)^8 \quad (4.9)$$

is also getting smaller, indicating that the anti D2-branes would likely spread out into a diffuse, as opposed to a thin, wall. More details regarding this analysis can be found in appendix B.

It is also interesting to note that when $\epsilon > 1/2$, the Maxwell charge at infinity

$$Q^{Maxwell} = Q + \frac{M^2}{2k} \quad (4.10)$$

flips sign. At this point, the stabilization radius r_* no longer exists for finite r . This appears to suggest that the system undergoes some kind of phase transition at $\epsilon = 1/2$. For the sake of illustration, we have drawn the fixed ϵ contours for $b_\infty = 1/2$ and for range of values $0 < \epsilon < \infty$ in logarithmic scale in figure 4. The plot is very similar to figure 12 in [9]. The phase transition at $\epsilon = 1/2$ is illustrated as the transition from the light red to the light green region.

In closing, let us also comment on a potentially interesting possibility of exploring the non-supersymmetric configuration corresponding to having $Q \ll -k < 0$ but having small

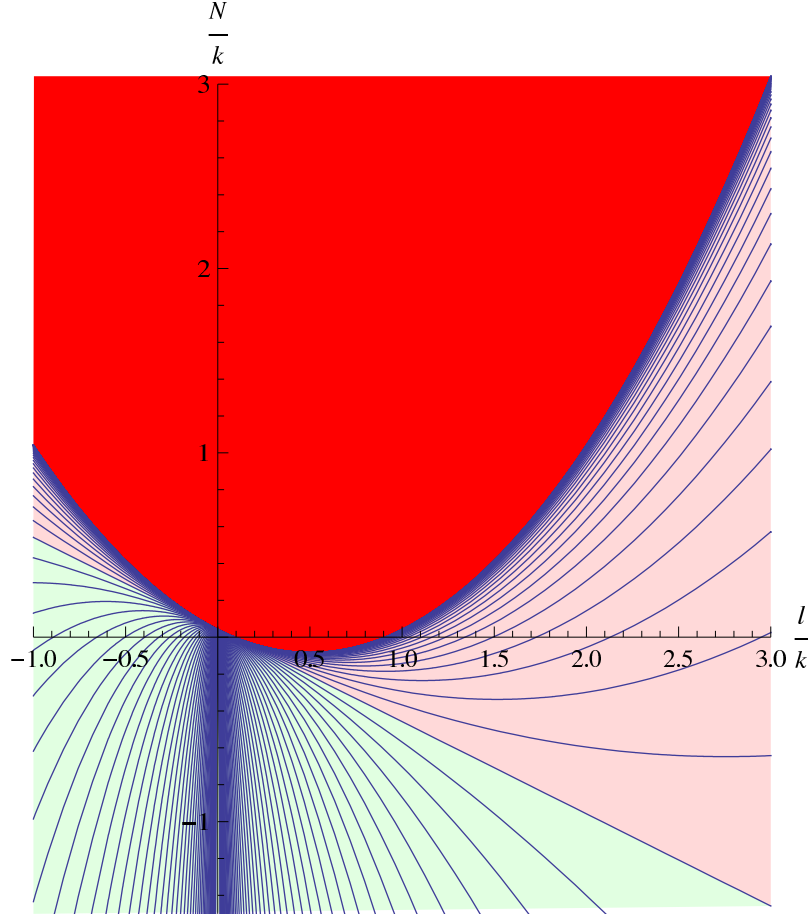


Figure 4: The phase diagram of the warped A_8 theory as a function of N/k and l/k . Here, we have set $b_\infty = 1/2$ and k is assumed to be large. The red parabola indicates the region where $Q > 0$ and the theory flows to the superconformal fixed point of ABJM. Outside the red parabola, we illustrate the contours of fixed ϵ in the range $0 < \epsilon < \infty$ in logarithmic intervals. At $\epsilon = 1/2$, the $Q_2^{Maxwell} = Q + M^2/2k$ changes sign, and we expect the theory to transition into a new phase as ϵ crosses this line. The supergravity approximation should be considered most reliable for large values of N/k and l/k and close to the red parabola corresponding to small values of ϵ .

M^2 which we treat as a perturbation. For $M^2 = 0$, the system should be described in supergravity by the standard anti D2 solution in A_8/Z_k with no self-dual 4-form turned on. Turning on a small self-dual 4-form will break supersymmetries incompatible with the anti D2-brane, and as such, leads to a non-supersymmetric solution. This can be explored either in the non-decoupled, i.e. for $H(r \rightarrow \infty) = 1$ solution, or the decoupled solution $H(r \rightarrow \infty) = 0$. It is relatively straightforward to set up the Borokhov-Gubser type analysis for this setup as well. The preliminary finding is that this expansion is much better behaved. It should be noted from the outset, that working in the regime $M^2 \ll -Qk$ is tantamount to working with $\epsilon \gg 1/2$ and so is deep in the region which we believe is in a different phase than the $\epsilon < 1/2$ region, as can be seen illustrated in figure 4. Nonetheless, this is part of the full landscape of possible parametric choices for these models and may teach us something interesting about non-supersymmetric dynamics of field theory and string theory.

Note Added

While this paper was in its final stages of preparation, a paper [41] appeared which has significant overlap on the analysis of the linearized supergravity equations. Our findings regarding the linearized analysis appear to be in complete agreement with [41].

Acknowledgements

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A D0 probe in $Q = 0$ background

In this appendix, we will briefly describe another brane probe one can consider for the $Q = 0$ background which exhibits interesting behavior.

Consider a probe D0-brane. One can compute the potential experienced by this probe simply using the DBI action

$$V(x) = \frac{1}{2\pi l_s g_s} e^{-\phi} \sqrt{g_{00}^{IIA}} = \frac{1}{2\pi l_s g_s} H(x)^{-1/2} b(x)^{-1} \quad (\text{A.1})$$

where $H(x)$ and $b(x)$ are given in (2.10) and (2.2), and

$$r = \ell x \quad (\text{A.2})$$

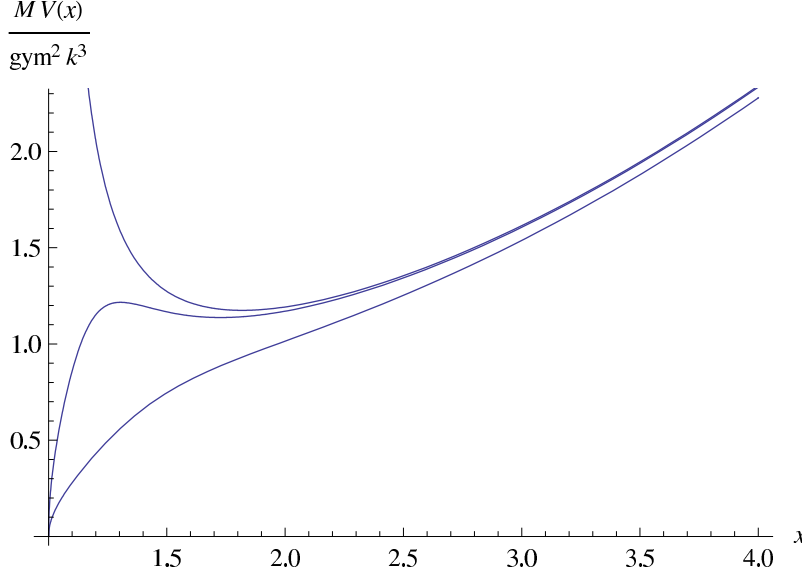


Figure 5: The potential of the D0-brane probe for $Q > 0$ background inferred from the DBI action. At $Q = 0$, the D0 brane is stabilized at a finite radius. For small and positive Q , the minimum at finite radius becomes metastable. As Q is increased, the metastable minimum disappears and the D0 is attracted toward the core region at $r = \ell$.

For $Q \gg 0$, the D0 probe action is attractive for all x . However, for $Q = 0$, there is a repulsive component to the potential, giving rise to a potential illustrated in figure 5.

When small positive Q is turned on, this potential exhibits a metastable minimum until Q reaches a critical value, at which point the metastable minimum goes away.

In the perspective of gauge gravity duality, this is an object which behaves as a localized magnetic flux which is stable and finite in size. In some respects, this is a IIA version of the axion string discussed in [42]. It might be interesting to further explore the physics of this object.

B Anti D2-brane probe in a repulson background

In this appendix, we provide the details of the anti D2 brane probe analysis in the repulson background. The anti D2 action takes the form

$$V(r) = T e^{-\phi} \sqrt{\det g_{\mu\nu}} = -T H^{-1}(r) \quad (\text{B.1})$$

for H given in (2.10), except that here, we also include the additional “1.”

The stable radius r_* for the anti D2-brane probe, given by

$$V'(r_*) = 0 \quad (\text{B.2})$$

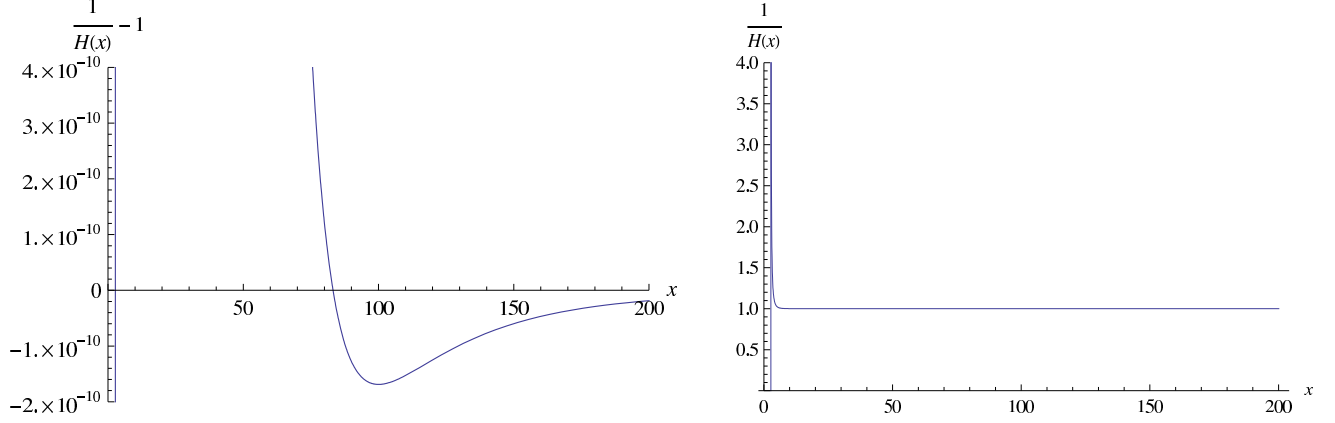


Figure 6: Anti D2 potential $H(x)^{-1}$ in the $Q = -k/24$ repulson background for the harmonic function $H(x)$ including the “1.” Here, we have set $g_s = k^{-1}$ and also set $x_* = 100$, which sets $M^2 \approx -2kQ$. The repulson radius is roughly at $x = 2.69$.

is equivalent to the condition that

$$H'(r_*) = 0 \quad (\text{B.3})$$

and so is insensitive to whether or not we include the “1.” Substituting (2.10), we find that this condition reads

$$0 = (\ell - r_*)^4 (123\ell^3 + 121r_*^2\ell^2 + 33r_*^2\ell + 3r_*^3) - 6\epsilon(\ell + r_*)^3(3\ell + r_*)^4 \quad (\text{B.4})$$

which can be used to solve for r_* in terms of ϵ , but it is just as convenient to solve for ϵ in terms of r_* . Then, we find

$$\epsilon = \frac{(\ell - r_*)^4 (123\ell^3 + 121r_*^2\ell^2 + 33r_*^2\ell + 3r_*^3)}{6(\ell + r_*)^3(3\ell + r_*)^4} = \frac{1}{2} - \frac{4\ell}{r_*} + \mathcal{O}((\ell/r_*)^2) \quad (\text{B.5})$$

Now, consider $V''(r_*)$. This expression depends critically on the “1” in the harmonic function. More specifically, this takes the form

$$V''(r = r_*) \sim \frac{T}{H(r = \infty)^2} \frac{\ell^8}{r_*^8} \sim \frac{T}{H(r = \infty)^2} (1 - 2\epsilon)^8 \quad (\text{B.6})$$

where we have only indicated the scaling with respect to r_* for large r_* as the expression is somewhat complicated. Nonetheless, the important point is that for $r_* \gg \ell$, i.e. for $\epsilon \lesssim 1/2$, it goes to zero very rapidly.

It may be useful to illustrate the anti D2-brane potential in the case $r_* \gg \ell$ more explicitly. In terms of

$$x = \frac{r}{\ell}, \quad x_* = \frac{r_*}{\ell} \quad (\text{B.7})$$

and for

$$Q = -\frac{k}{24} \quad (\text{B.8})$$

the harmonic function H can be written as

$$H(x) = 1 + \frac{\pi^2}{60g_s^4k^4} \left[-\frac{20(3x^3 - 3x^2 - 11x + 27)}{(x-1)^3(x+3)} - 15 \log\left(\frac{x-1}{x+3}\right) + \frac{768(x_*+1)^3(x_*+3)^4(3x^2+26x+63)}{(x_*-1)^4(3x_*^3+33x_*^2+121x_*+123)(x+1)^2(x+3)^5} \right] \quad (\text{B.9})$$

Note the presence of “1” in $H(x)$. A convenient choice to illustrate the possibility of separating r_* from the repulson radius is to chose $g_s = k^{-1}$ and $x_* = 100$. With this choice, the potential $V(x) \propto H(x)^{-1}$ has the form illustrated in figure 6. We have specifically included the same plot in two different scaling of the axes in order to highlight the features in very large and very small scales.

Suppose the idea that the BPS solution is reliable, at least for large radius, down to $Q = -k/24$ with the understanding that string dynamics self corrects the geometry in the $r < r_*$ region. What this example illustrates is that one should anticipate string corrections to impact regions all the way up to $r_* = 100\ell$ which is significantly further out than where one would have expected the corrections based merely on the estimate of the curvature which is concentrated in $r \approx \ell$ region. If correct, this would be a novel mechanism to induce larger corrections to gravity than what one would naively expect in effective field theory considerations.

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